

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT2

Branch : Diploma(All)

Semester : 2

Date : 04/05/2017

Time : 02:00 To 05:00

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

- Q-1 Attempt the following questions: (14)**
- a) If the two vectors \vec{x} and \vec{y} are orthogonal to each other then $\vec{x} \cdot \vec{y} = \underline{\hspace{2cm}}$. **(1)**
(a) 0 (b) 1 (c) -1 (d) none of these
- b) If $\vec{a} = i + j + k$ and $\vec{b} = i + 0j + 0k$ then $\vec{a} - \vec{b} = \underline{\hspace{2cm}}$. **(1)**
(a) (-1, 4, -2) (b) (-1, 4, 2) (c) (1, 4, -2) (d) none of these
- c) $|(2, 1, 1) + (1, 2, 3)| = \underline{\hspace{2cm}}$ **(1)**
(a) $\sqrt{17}$ (b) $2\sqrt{17}$ (c) 34 (d) $\sqrt{34}$
- d) If $f(x) = \sin x$ then $f\left(\frac{\pi}{2}\right) = \underline{\hspace{2cm}}$ **(1)**
(a) -1 (b) 0 (c) 1 (d) 2
- e) $\lim_{x \rightarrow 0} \left(1 + \frac{2}{x}\right)^x = \underline{\hspace{2cm}}$ **(1)**
(a) e^3 (b) e (c) $e^{1/3}$ (d) none of these
- f) $\lim_{x \rightarrow 0} \frac{x^3 - 2^3}{x - 2} = \underline{\hspace{2cm}}$ **(1)**
(a) $3(2)^{3-1}$ (b) $3(2)^{3-2}$ (c) $4(2)^{2-1}$ (d) none of these
- g) $\lim_{x \rightarrow 0} \frac{x^2 + x - 2}{x - 1} = \underline{\hspace{2cm}}$ **(1)**
(a) 1 (b) 3 (c) -1 (d) none of these
- h) $\frac{d\left(\frac{1}{x}\right)}{dx} = \underline{\hspace{2cm}}$ **(1)**
(a) $\log x$ (b) $-\frac{1}{x}$ (c) $\frac{1}{x^2}$ (d) $\frac{-1}{x^2}$



- i) $\frac{d(\cot x)}{dx} = \underline{\hspace{2cm}}$ (1)
 (a) $\operatorname{cosec}^2 x$ (b) $-\operatorname{cosec}^2 x$ (c) $\sec^2 x$ (d) $-\sec^2 x$
- j) $\frac{d(\cos^{-1} x)}{dx} = \underline{\hspace{2cm}}$ (1)
 (a) $\frac{-1}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{1+x^2}$ (d) none of these
- k) $\int a^x dx = \underline{\hspace{2cm}}$ (1)
 (a) $a^x \log_e a + c$ (b) $\frac{\log_e a}{a^x} + c$ (c) $\frac{a^x}{\log_e a} + c$ (d) none of these
- l) $\int \cos x dx = \underline{\hspace{2cm}}$ (1)
 (a) $\sec x + c$ (b) $-\sec x + c$ (c) $-\sin x + c$ (d) $\sin x + c$
- m) $\int \frac{1}{\sqrt{1-x^2}} dx = \underline{\hspace{2cm}}$ (1)
 (a) $\sin^{-1} x + c$ (b) $-\sin^{-1} x + c$ (c) $\cos^{-1} x + c$ (d) $-\cos^{-1} x + c$
- n) $\int_0^1 e^x dx = \underline{\hspace{2cm}}$ (1)
 (a) $1-e$ (b) $e-1$ (c) e (d) none of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

- a) If $\bar{a} = 3i - j - 4k$, $\bar{b} = -2i + 4j - 3k$ and $\bar{c} = -i + 2j - 5k$ then find $|\bar{a} + 2\bar{b} - \bar{c}|$. (5)
- b) Evaluate: $\lim_{x \rightarrow a} \frac{\sqrt{2a-x} - \sqrt{x}}{a-x}$ (5)
- c) Find $\frac{dy}{dx}$ if $y = \frac{1+\sin x}{1-\sin x}$. (4)

Q-3 Attempt all questions (14)

- a) Prove that the angle between two vectors $i + j - k$ and $2i - 2j + k$ is $\sin^{-1}\left(\frac{\sqrt{26}}{3\sqrt{3}}\right)$. (5)
- b) Using definition find derivative of $f(x) = x^3$. (5)
- c) Find $\int \frac{\cos \sqrt{x}}{2\sqrt{x}} dx$. (4)

Q-4 Attempt all questions (14)

- a) Evaluate: $\lim_{x \rightarrow 0} \frac{2(5)^x + 3(2)^x - 5}{x}$ (5)



b) Find $\frac{dy}{dx}$ if $y = \log(\sec x + \tan x)$. (5)

c) For what value of m , vectors $2i + mj + k$ and $2i + 4j + 5k$ are perpendicular to each other? (4)

Q-5 Attempt all questions (14)

a) Find $\frac{dy}{dx}$ at $t = 3$ if $x = \frac{a}{2}\left(t + \frac{1}{t}\right)$ and $y = \frac{b}{2}\left(t - \frac{1}{t}\right)$. (5)

b) Find $\int x e^x dx$. (5)

c) Evaluate: $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 3}{x^2 + 2x - 3}$ (4)

Q-6 Attempt all questions (14)

a) Forces $\vec{F}_1 = 3i - j + 2k$ and $\vec{F}_2 = i + 3j - k$ act on a particle under the influence of these forces, particle moves from the point $(2, 3, 1)$ to $(5, 2, 3)$. Find the work done. (5)

b) Prove that $\int_0^{\pi/2} \frac{\tan x}{\tan x + \cot x} dx = \frac{\pi}{4}$. (5)

c) The equation of motion of a particle is $s = -5t^3 + 15t + 3$. Find velocity and acceleration after 3 seconds. (4)

Q-7 Attempt all questions (14)

a) If $f'(x) = 4x^2 + 6x - 3$ and $f(1) = 2$ then find function $f(x)$. (5)

b) Find $\frac{dy}{dx}$ if $y = (\sin x)^x$. (5)

c) Find unit vector which is perpendicular to $\vec{x} = 5i + 7j - 2k$ and $\vec{y} = 3i + j - 2k$. (4)

Q-8 Attempt all questions (14)

a) Find area of region bounded between $y = x^2$, X-axis, $x = 1$ and $x = 2$. (5)

b) If $\vec{a} = (2, -1, 0)$ and $\vec{b} = (1, 3, -2)$ then find $\left|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})\right|$. (5)

c) If $f(x) = e^x$ then prove that (i) $f(x + y) = f(x)f(y)$ (ii) $f(x - y) = \frac{f(x)}{f(y)}$. (4)

